

A-LEVEL Mathematics

Further Pure3 – MFP3 Mark scheme

6360 June 2015

Version/Stage: Final Mark Scheme V1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Μ	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
А	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and
	accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

Key to mark scheme abbreviations

manule in far manth ad

N /

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment		
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION					
(a)	$y(2.05) = y(2) + 0.05\left(\frac{2+5^2}{2}\right)$	M1				
	$= 5 + 0.05 \times 13.5$					
	= 5.675	A1	2	OE		
(b)	$y(2.1) = y(2) + 2 \times 0.05 \text{ f}[2.05, y(2.05)]$	M1				
	$= 5 + 2 \times 0.05 \times \left(\frac{2.05 + 5.675^2}{2.05}\right)$	A1F		PI Ft on c's (a) answer.		
	= 6.67 to 3 sf	A1	3	CAO Must be 6.67		
	Total		5			
	(b) For the PI if line missing, check to see i	f evaluati	on match	es $5.1 + \frac{2}{41} \times [\text{answer}(a)]^2$ to at least 3sf		

Q2	Solution	Mark	Total	Comment
	$\int \tan x dx$	M1		
	I.F. e			
	$= e^{\ln \sec x}$	A1		OE eg $e^{-\ln \cos x}$
	$= \sec x$	A1F		OE Only ft sign error in integrating $\tan x$.
	$\sec x \frac{dy}{dx} + \sec x(\tan x)y = \tan^3 x \sec^2 x$ $\frac{d}{dx} [y \sec x] = \tan^3 x \sec^2 x$	M1		LHS as $\frac{d}{dx}[y \times \text{candidate's IF}]$ PI
	$dx = \int \tan^3 x \sec^2 x \ (dx)$	A1		dx ¹²
	$y \sec x = \int t^3 dt$	m1		PI OE eg y sec $x = \int \left(\frac{1}{u^3} - \frac{1}{u^5}\right) du$,
	$y \sec x = \frac{1}{4} \tan^4 x \ (+c)$	A1		where $u = \cos x$
	$2\sec\frac{\pi}{3} = \frac{1}{4}\tan^4\frac{\pi}{3} + c; 4 = \frac{9}{4} + c$	m1		Dep on prev MMm. Correct boundary condition applied to obtain an eqn in <i>c</i>
				with correct exact value for either $\sec \frac{\pi}{3}$ or
				$\tan^4 \frac{\pi}{3}$ used
	$y \sec x = \frac{1}{4} \tan^4 x + \frac{7}{4}$			
	$y = \frac{\cos x}{4} \left(7 + \tan^4 x\right)$	A1	9	ACF
	Total 9			
	Condone answer left in a 'correct' form different to $y = f(x)$, eg $4y \sec x = \tan^4 x + 7$.			

Q3	Solution	Mark	Total	Comment
(a)(i)	$\ln(1+2x) = 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} \dots$ $= 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 \dots$	B1	1	ACF Condone correct unsimplified
(a)(ii)	$\ln[(1+2x)(1-2x)] = \ln(1+2x) + \ln(1-2x)$	M1		$\ln(1+2x) + \ln(1-2x) \text{ PI}$ {or $\ln(1-4x^2) = -4x^2 - \frac{(-4x^2)^2}{2} \dots$ } PI
	$= -4x^2 - 8x^4 \dots$ Expansion valid for $-\frac{1}{2} < x < \frac{1}{2}$	A1 B1	3	CSO Must be simplified Condone $ x < \frac{1}{2}$
(b)	$x\sqrt{9+x} = 3x\left[1 + \frac{x}{18} + O(x^2)\right]$	B1		Correct first two terms in expn. of $\sqrt{9+x}$
	$\left[\frac{3x - x\sqrt{9 + x}}{\ln[(1 + 2x)(1 - 2x)]}\right] = \left[\frac{3x - 3x - \frac{3x^2}{18}}{-4x^2 - 8x^4}\right]$	M1		Series expansions used in both numerator and denominator.
	$\lim_{x \to 0} \left[\frac{3x - x\sqrt{9 + x}}{\ln[(1 + 2x)(1 - 2x)]} \right]$			
	$=\lim_{x \to 0} \left[\frac{-\frac{1}{6} + O(x)}{-4 + O(x^2)} \right]$	m1		Dividing numerator and denominator by x^2 to get constant term in each, leading to a finite limit. Must be at least a total of 3 'terms' divided by x^2
	$=\frac{1}{24}$	A1	4	$=\frac{1}{24}$ NOT $\rightarrow \frac{1}{24}$
	Total		8	

Q4	Solution	Mark	Total	Comment
(a)	The interval of integration is infinite	E 1	1	OE
(b)	$\int (x-2)e^{-2x} dx$ $u = x - 2, \frac{dv}{dx} = e^{-2x}, \frac{du}{dx} = 1, v = -0.5e^{-2x}$ $\dots = -\frac{1}{2}(x-2)e^{-2x} - \int -\frac{1}{2}e^{-2x} dx$ $= -\frac{1}{2}(x-2)e^{-2x} - \frac{1}{4}e^{-2x} (+c)$	M1 A1 A1		$\frac{du}{dx} = 1, v = k e^{-2x} \text{ with } k = \pm 0.5, \pm 2$ $-\frac{1}{2}(x-2)e^{-2x} - \int -\frac{1}{2}e^{-2x} (dx) \text{ OE}$
	2 4	AI		
	$\int_{2}^{\infty} (x-2) e^{-2x} dx = \lim_{a \to \infty} \int_{2}^{a} (x-2) e^{-2x} dx$	M1		Evidence of limit ∞ having been replaced by <i>a</i> (OE) at any stage and $\lim_{a \to \infty}$ seen or taken at any stage with no remaining lim relating to 2.
	$\lim_{a \to \infty} \left[-\frac{1}{2} (a-2) e^{-2a} - \frac{1}{4} e^{-2a} \right] - \left(-\frac{1}{4} e^{-4} \right)$			
	Now $\lim_{a \to \infty} a^p e^{-2a} = 0$, (p>0)	E 1		General statement or specific statement with $p = 1$ stated explicitly. Each must include the 2 in the exponential.
	$\int_{2}^{\infty} (x-2) e^{-2x} dx = \frac{1}{4} e^{-4}$	A1	6	No errors seen in $F(a) - F(2)$. (M1E0A1 is possible)
	Total		7	

(a)Aux eqn $m^2 + 6m + 9 = 0$ $(m+3)^2 = 0$ M1Factorising or using quadratic formula OE on correct aux eqn. PI by correct value of 'm' seen/used. $(y_{CF} =) (Ax + B)e^{-3x}$ A1Try $(y_{PI} =) a \sin 3x + b \cos 3x$ M1 $(y'_{PI} =) 3a \cos 3x - 3b \sin 3x$ M1 $(y'_{PI} =) -9a \sin 3x - 9b \cos 3x$ M1 $-9a \sin 3x - 9b \cos 3x + 6(3a \cos 3x - 3b \sin 3x)$ m1 $+9(a \sin 3x + b \cos 3x) = 36 \sin 3x$ m1 $-18b = 36$ $18a = 0$ $y_{PI} = -2\cos 3x$ $-18b = 36$ $18a = 0$ $y_{PI} = -2\cos 3x$ A1A1	Q5	Solution	Mark	Total	Comment
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $					Factorising or using quadratic formula OE
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			M1		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $. ,	A1		m seen/used.
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $					$a\sin 3x \pm b\cos 3x$ or Alth $k\cos 3x$
$ \begin{pmatrix} y'_{P_{I}} =) - 9a \sin 3x - 9b \cos 3x \\ - 9a \sin 3x - 9b \cos 3x + 6(3a \cos 3x - 3b \sin 3x) \\ + 9(a \sin 3x + b \cos 3x) = 36 \sin 3x \\ \end{pmatrix} $ m1 $ \begin{array}{c} Substitution into DE, dep on previous M \\ and differentiations being \\ in form p \cos 3x + q \sin 3x \\ or Altn 3k \sin 3x and - 9k \cos 3x \\ Seen or used \\ Correct y_{P_{I}} seen or used \\ Correct y_{P_{I}} seen or used \\ (y_{GS} =) (Ax + B)e^{-3x} - 2\cos 3x \\ f''(0) + 6f'(0) + 9f(0) = 36 \sin 0 \\ f''(0) + 6(0) + 9(0) = 0 \implies f''(0) = 0 \\ f''(0) = 108 \cos 0 - 0 = 108 \\ f(x) \approx 0 + x(0) + \frac{x^{2}}{2} (0) + \frac{x^{3}}{3!} f'''(0) + \frac{x^{4}}{4!} f^{(iv)}(0) \dots \\ f(x) \approx \frac{x^{3}}{3!} (108) + \frac{x^{4}}{4!} (- 648) \dots \\ = 18x^{3} - 27x^{4} \\ Aln \\ \frac{Alm:}{1} Use of answer to part (a) \\ f(x) = (6x + 2)e^{-3x} - 2\cos 3x \\ = \\ \begin{array}{c} (B1) \\ (B1) \\ = \\ (2-2) + (6-6)x + (9-18+9)x^{2} + (27-9)x^{3} + \\ + (6.75-27-6.75)x^{4} \\ = 18x^{3} - 27x^{4} \\ \end{array} [A1][A$			1411		$a \sin 5x + b \cos 5x$ of Atti. $k \cos 5x$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $					
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $					Substitution into DE dan on provious M
$\begin{array}{c c c c c c c c c c c c c c c c c c c $			m1		
$ \begin{vmatrix} -18b = 36 & 18a = 0 \\ y_{Pl} = -2\cos 3x & A1 \\ (y_{GS} =) & (Ax + B)e^{-3x} - 2\cos 3x & B1F \\ (y_{GS} =) & (Ax + B)e^{-3x} - 2\cos 3x & B1F \\ (y_{GS} =) & (Ax + B)e^{-3x} - 2\cos 3x & B1F \\ (y_{GS} =) & (Ax + B)e^{-3x} - 2\cos 3x & B1F \\ (y_{GS} =) & (Ax + B)e^{-3x} - 2\cos 3x & B1F \\ (y_{GS} =) & (Cx + C^* S P1, must have exactly two arbitrary constants & P1^{(0)}(0) = 0 & CT + C^* S P1, must have exactly two arbitrary constants & P1^{(0)}(0) = 0 & CT + C^* S P1, must have exactly two arbitrary constants & P1^{(0)}(0) = 0 & CT + C^* S P1, must have exactly two arbitrary constants & P1^{(0)}(0) = 0 + G1^{(0)}(0) = 0 & CT + C^* S P1, must have exactly two arbitrary constants & P1^{(0)}(0) = 0 + G1^{(0)}(0) = 0 + G1^{$		$+9(a \sin 3x + b \cos 3x) = 36 \sin 3x$			-
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $					
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		-18b = 36 $18a = 0$			Seen or used
(b)(i) $f''(0) + 6f'(0) + 9f(0) = 36 \sin 0$ $f''(0) + 6(0) + 9f(0) = 36 \sin 0$ $f''(0) + 6(0) + 9(0) = 0 \implies f''(0) = 0$ (b)(ii) $f'''(0) = 108 \cos 0 - 0 = 108$ $f^{(iv)}(0) = 0 - 6f'''(0) - 0 = -648$ $f(x) \approx 0 + x^2 - (0) + \frac{x^2}{3!} f'''(0) + \frac{x^4}{4!} f^{(iv)}(0) \dots$ $f(x) \approx \frac{x^3}{3!} (108) + \frac{x^4}{4!} (-648) \dots$ $= 18x^3 - 27x^4$ A1 A1 A1 A1 A1 A1 A1 A1 A1 A1		$y_{PI} = -2\cos 3x$	A1		Correct y_{PI} seen or used
(b)(i) $f''(0) + 6f'(0) + 9f(0) = 36 \sin 0$ $f''(0) + 6(0) + 9(0) = 0 \implies f''(0) = 0$ (b)(ii) $f''(0) = 108 \cos 0 - 0 = 108$ $f^{(iv)}(0) = 0 - 6f'''(0) - 0 = -648$ $f(x) \approx 0 + x(0) + \frac{x^2}{2}(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(iv)}(0)$ $f(x) \approx \frac{x^3}{3!}(108) + \frac{x^4}{4!}(-648)$ $= 18x^3 - 27x^4$ A1 A1 A1 A1 A1 A1 A1 A1 A1 A1		$(y_{GS} =) (Ax + B)e^{-3x} - 2\cos 3x$	B1F	_	$(y_{GS} =)$ c's CF + c's PI, must have exactly
$ \begin{aligned} f''(0) + 6(0) + 9(0) = 0 \implies f''(0) = 0 \\ f''(0) = 108 \cos 0 - 0 = 108 \\ f^{(iv)}(0) = 0 - 6 f'''(0) - 0 = -648 \\ f(x) \approx 0 + x(0) + \frac{x^2}{2}(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{(iv)}(0) \dots \\ f(x) \approx \frac{x^3}{3!}(108) + \frac{x^4}{4!}(-648) \dots \\ &= 18x^3 - 27x^4 \\ Aln: Use of answer to part (a) \\ f(x) = (6x + 2)e^{-3x} - 2\cos 3x \\ &= (2-2) + (6-6)x + (9-18+9)x^2 + (27-9)x^3 + \\ &+ (6.75-27-6.75)x^4 \\ &= 18x^3 - 27x^4 \\ \hline \\ $	<i></i>			7	two arbitrary constants
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	(b)(I)		F1	1	AG Convincingly shown with no errors
f ^(iv) (0) = 0-6 f'''(0) -0 = -648 B1 $f(x) \approx 0 + x(0) + \frac{x^2}{2}(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(iv)}(0) \dots$ M1 $f(x) \approx \frac{x^3}{3!}(108) + \frac{x^4}{4!}(-648)\dots$ M1 $= 18x^3 - 27x^4$ A1 Altn: Use of answer to part (a) [B1] $f(x) = (6x + 2)e^{-3x} - 2\cos 3x$ [B1] $= (2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3 + +(6.75-27-6.75)x^4$ [A1] $= 18x^3 - 27x^4$ [A1] [3] Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms inclusive) and $\cos 3x$ (at least x^2 terms and x^4 terms) substituted and also product of $(px+q)$ term with e^{-3x} series attempted where p and q are numbers.		$f''(0) + 6(0) + 9(0) = 0 \implies f''(0) = 0$	171	L	AC Convincingly shown with no criors.
f ^(iv) (0) = 0-6 f'''(0) -0 = -648 B1 $f(x) \approx 0 + x(0) + \frac{x^2}{2}(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(iv)}(0) \dots$ M1 $f(x) \approx \frac{x^3}{3!}(108) + \frac{x^4}{4!}(-648)\dots$ M1 $= 18x^3 - 27x^4$ A1 Altn: Use of answer to part (a) [B1] $f(x) = (6x + 2)e^{-3x} - 2\cos 3x$ [B1] $= (2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3 + +(6.75-27-6.75)x^4$ [A1] $= 18x^3 - 27x^4$ [A1] [3] Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms inclusive) and $\cos 3x$ (at least x^2 terms and x^4 terms) substituted and also product of $(px+q)$ term with e^{-3x} series attempted where p and q are numbers.	(b)(ii)	$f''(0) = 108_{000} - 0 - 0 = 108$			$f'''(0) = 109$ and $f^{(iv)}(0) = -649$ scorp or used
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	(0)(11)		B1		1 (0) - 108 and 1 (0)048 seen of used
$f(x) \approx \frac{x^3}{3!} (108) + \frac{x^4}{4!} (-648) \dots$ $= 18x^3 - 27x^4$ A1 $\frac{A1}{f(x)} \approx \frac{x^3}{3!} (108) + \frac{x^4}{4!} (-648) \dots$ $= 18x^3 - 27x^4$ A1 $\frac{A1}{f(x)} = (6x + 2)e^{-3x} - 2\cos 3x$ $= [M1]$ $= [M1]$ $= [M1]$ $= (2-2) + (6-6)x + (9-18+9)x^2 + (27-9)x^3 + + (6.75-27-6.75)x^4]$ $= 18x^3 - 27x^4$ [A1] [A1] [A1] [A1] [A1] [A1] [A1] [A1]					x^3 x^4 (iii)
$f(x) \approx \frac{x}{3!} (108) + \frac{x}{4!} (-648) \dots$ $= 18x^3 - 27x^4$ A1 3 $\frac{Altn:}{f(x)} = (6x + 2)e^{-3x} - 2\cos 3x$ $= 18x^3 - 2\cos 3x$ $= 18x^3 - 27x^4$ $[B1]$ $[M1]$ $A1$ 3 $18x^3 - 27x^4$ $[B1]$ $[M1]$ $Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms) substituted and also product of (px+q) term with e^{-3x} series attempted where p and q are numbers. = (2-2) + (6-6)x + (9-18+9)x^2 + (27-9)x^3 + (6.75-27-6.75)x^4 = 18x^3 - 27x^4 [A1] [A1] [3]$		$f(x) \approx 0 + x(0) + \frac{1}{2}(0) + \frac{1}{3!}f''(0) + \frac{1}{4!}f''(0)$	M1		$f(x) \approx \frac{1}{3!} f''(0) + \frac{1}{4!} f''(0)$ used with c's
$= 18x^{3} - 27x^{4}$ $= 18x^{3} - 27x^{4}$ $= 18x^{3} - 27x^{4}$ $\frac{\text{A1}}{\text{f(x)} = (6x + 2)e^{-3x} - 2\cos 3x}$ $= 18x^{3} - 2\cos 3x$ $= (2-2) + (6-6)x + (9-18+9)x^{2} + (27-9)x^{3} + (6.75-27-6.75)x^{4} + (6.75-27-6.75)x^{4}$ $= 18x^{3} - 27x^{4}$ $= 11x^{3} - 27x^{4}$		$f(x) \approx \frac{x^3}{(108)} + \frac{x^4}{(108)}$			non-zero values for $f''(0)$ and $f^{(iv)}(0)$
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$1(x) \approx \frac{1}{3!}(108) + \frac{1}{4!}(-648)$			
Altn:Use of answer to part (a) $f(x) = (6x+2)e^{-3x} - 2\cos 3x$ [B1]=[M1][M1]=[M1]Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms) substituted and also product of $(px+q)$ term with e^{-3x} series attempted where p and q are numbers.=(2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3+ +(6.75-27-6.75)x^4[A1][3]Image: Alternative conduction of the series of the series of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series attempted where p and q are numbers.[A1]Image: Alternative conduction of the series attempted where p and q are numbers.[A1]		$= 18x^3 - 27x^4$	A1	•	$18x^3 - 27x^4$ Ignore any extra higher
$f(x) = (6x+2)e^{-3x} - 2\cos 3x$ $[B1]$ $=$ $[M1]$ $[M1]$ $Correct series for e^{-3x} (at least from x^{2} terms up to x^{4} terms) substituted and also product of (px+q) term with e^{-3x} series attempted where p and q are numbers. =(2-2)+(6-6)x+(9-18+9)x^{2}+(27-9)x^{3}+ +(6.75-27-6.75)x^{4} = 18x^{3} - 27x^{4} [A1] [3]$				3	powers of <i>x</i> terms
$= \begin{bmatrix} 1(x) - (0x + 2)c & 2\cos 3x \\ = \end{bmatrix} \begin{bmatrix} M1 \end{bmatrix}$ $\begin{bmatrix} M1 \end{bmatrix}$ $\begin{bmatrix} Correct series for e^{-3x} (at least from x^{2} terms up to x^{4} terms inclusive) and cos 3x (at least x^{2} terms and x^{4} terms) substituted and also product of (px+q) term with e^{-3x} series attempted where p and q are numbers. = (2-2) + (6-6)x + (9-18+9)x^{2} + (27-9)x^{3} + (6.75-27-6.75)x^{4} = 18x^{3} - 27x^{4} \begin{bmatrix} A1 \end{bmatrix} \begin{bmatrix} 3 \end{bmatrix}$		-	[R 1]		
$=(2-2)+(6-6)x+(9-18+9)x^{2}+(27-9)x^{3}++(6.75-27-6.75)x^{4}$ $=18x^{3}-27x^{4}$ [A1] [3] [3] [41] [3] [3] [3]					Connect coming for e^{-3x} (at least from e^2
$=(2-2)+(6-6)x+(9-18+9)x^{2}+(27-9)x^{3}++(6.75-27-6.75)x^{4}$ $= 18x^{3}-27x^{4}$ [A1] [3] [41] [3] [41] [3]		=			
$=(2-2)+(6-6)x+(9-18+9)x^{2}+(27-9)x^{3}++(6.75-27-6.75)x^{4}$ $= 18x^{3} - 27x^{4}$ [A1] [3] [3] [3]					(at least x^2 terms and x^4 terms) substituted
$=(2-2)+(6-6)x+(9-18+9)x^{2}+(27-9)x^{3}++(6.75-27-6.75)x^{4}$ $= 18x^{3} - 27x^{4}$ [A1] [3] numbers.					
$ \begin{array}{c c} =(2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3+\\ +(6.75-27-6.75)x^4\\ = 18x^3-27x^4 \end{array} \qquad [A1] \qquad [3] \\ \hline $					
Total 11		$=(2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3+$			
Total 11		$+(6.75-27-6.75)x^4$			
		$= 18x^3 - 27x^4$	[A1]	[3]	
		Total		11	
In using (a) to answer (b)(1), for guidance, $T(x) = -34xe^{-1} = 10e^{-1} + 100083x$			$-54 r c^{-3}$		$+18\cos 3r$
		In using (a) to answer (b)(i), for guidance, $T(x)$	- J41C	-100	T 100053A

Q6	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}$			OE Relevant chain rule eg $\frac{dy}{dx} = \frac{dt}{dx} \frac{dy}{dt}$
		M1		$\frac{\partial U}{\partial t} = \frac{\partial U}{\partial t} = $
	$2e^{2t} \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow 2x\frac{dy}{dx} = \frac{dy}{dt}$	A1		OE eg $\frac{dy}{dx} = \frac{1}{2}e^{-2t}\frac{dy}{dt}$
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(2x\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}; \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$	M1		OE. Valid 1 st stage to differentiate $xy'(x)$ oe wrt <i>t</i> or to differentiate $x^{-1}y'(t)$ oe wrt <i>x</i> .
	$\frac{\mathrm{d}x}{\mathrm{d}t} \left(2\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \right) = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$	m1		Product rule OE (dep on MM) to obtain an eqn involving both second derivatives
	$4x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 4x \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$	A1		OE eg $\frac{d^2 y}{dx^2} = \frac{1}{2} e^{-2t} \left[-e^{-2t} \frac{dy}{dt} + \frac{1}{2} e^{-2t} \frac{d^2 y}{dt^2} \right]$
				{Note: e^{-t} could be replaced by $\frac{1}{\sqrt{x}}$ }
	$4\sqrt{x^5} \frac{d^2 y}{dx^2} + 2\sqrt{x} \ y = \sqrt{x} (\ln x)^2 + 5$			
	becomes $\frac{d^2 y}{dt^2} - 4x \frac{dy}{dx} + 2y = (\ln x)^2 + \frac{5}{\sqrt{x}}$	A1		Or better
	$\Rightarrow \frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + 2y = (2t)^2 + \frac{5}{e^t}$			
	$\Rightarrow \frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + 2y = 4t^2 + 5e^{-t}$	A1	7	AG Be convinced
(b)	Auxl eqn $m^2 - 2m + 2 = 0$ $(m-1)^2 + 1 = 0$	M1		$(m-1)^2 + k$ or using quadratic formula on correct aux eqn. PI by correct values of 'm' seen/used.
	$m = 1 \pm i$	A1 D1E		
	CF: $(y_c =) e^t (A \cos t + B \sin t)$	B1F		Ft on $m = p \pm qi$, $p, q \neq 0$ and 2 arb.
	P.Int. Try $(y_P =) a + bt + ct^2 + de^{-t}$	M1		constants in CF. Condone x for t here
	(y '(t)=) $b + 2ct - de^{-t}$; (y ''(t)=) $2c + de^{-t}$ Substitute into DE gives			
	$2c + de^{-t} - 2(b + 2ct - de^{-t}) + + 2(a + bt + ct^{2} + de^{-t}) = 4t^{2} + 5e^{-t}$	M1		Substitution and comparing coeffs at least once
	d = 1; c = 2	B 1		Need both
	2b - 4c = 0 and $2c - 2b + 2a = 0$	A1		OE PI by c's $b=2\times$ c's c and c's $a=$ c's c provided c's $c\neq 0$
	b = 4 and $a = 2$	A1		Need both
	GS (y=) $e^{t}(A\cos t + B\sin t) + 2 + 4t + 2t^{2} + e^{-t}$	B1F		Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of <i>t</i> only
	$y = \sqrt{x} \left[A \cos\left(\ln \sqrt{x}\right) + B \sin\left(\ln \sqrt{x}\right) \right] + 2 + 2 \ln x + \frac{1}{2} (\ln x)^2 + \frac{1}{\sqrt{x}}$	A1	10	y=f(x) with ACF for $f(x)$
	-			
	Total		17	

Q7	Solution	Mark	Total	Comment	
(a)					
	Area = $\frac{1}{2} \int_{(-\frac{\pi}{2})}^{(\frac{\pi}{2})} (1 + \cos 2\theta)^2 (d\theta)$	M1		Use of $\frac{1}{2}\int r^2 (d\theta)$ or $\int_0^{\frac{\pi}{2}} r^2 (d\theta)$	
	$1 \cdot \frac{\pi}{2}$				
	$= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + 2\cos 2\theta + \cos^2 2\theta\right) d\theta$	B1		Correct expn of $[1+\cos 2\theta]^2$ and correct	
				limits	
	$=\frac{1}{2}\int (1+2\cos 2\theta+0.5+0.5\cos 4\theta)\mathrm{d}\theta$	M1		$2\cos^2 2\theta = \pm 1 \pm \cos 4\theta$ used with $k \int r^2 d\theta$	
	$=\frac{1}{2}\theta + \sin 2\theta + 0.5\theta + \frac{1}{8}\sin 4\theta \bigg _{-\pi/2}^{\pi/2}$	A1F		Correct integration ft wrong coefficients	
	$=\frac{3}{4}\pi$	A1	5	CSO	
(b)(i)	$1 + \sin \theta = 1 + \cos 2\theta$			Equating r_s (or equating $\sin\theta_s$) followed	
	$1 + \sin \theta = 1 + 1 - 2\sin^2 \theta$	M1		(or preceded) by $\cos 2\theta = \pm (1 \pm 2\sin^2 \theta)$	
	$(2\sin\theta - 1)(\sin\theta + 1)$ (=0)	A1		Or $r(2r-3)$ (=0), each PI by correct 2 roots	
	$\sin \theta = -1$ gives the pole, <i>O</i>	E1		Or $r = 0$ gives the pt O. OE eg finds 2^{nd}	
	(3π)			pair of coords $(0, -\pi/2)$ and chooses $(3/2, \pi/6)$	
	At A, $\sin \theta = 0.5 \operatorname{so} A\left(\frac{3}{2}, \frac{\pi}{6}\right)$	A1	4	$r=1.5, \ \theta=\frac{\pi}{6}$	
(b)(ii)	Eqn of line thro' A parallel to initial line is	DIE			
	$r\sin\theta = \frac{3}{4}$	B1F		PI Ft on $r \sin \theta = r_A \sin \theta_A$	
		M1		Solving $r \sin \theta = k$ and $r = 1 + \cos 2\theta$ to	
	At <i>B</i> , $r = 2 - 2\sin^2 \theta = 2 - 2\left(\frac{9}{16r^2}\right)$			reach a cubic eqn in r or in sin θ	
	$16r^3 = 32r^2 - 18$	A1		Correct cubic eqn in <i>r</i>	
	$(2r-3)(4r^2-2r-3)=0$	A1		(or in $\sin\theta \text{eg} 8\sin^3\theta = 8\sin\theta - 3$)	
		AI		Or $(2\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 3) = 0$ A.G. Note: A2 requires correct surd for <i>OB</i> and also	
	Since $r_A = 1.5$ and $r_B > 0$,			correct justifications for ignoring the other two roots	
	$OB = r_B = \frac{2 + \sqrt{4 + 48}}{8} = \frac{1}{4} \left(\sqrt{13} + 1 \right)$	A2,1,0	6	of the cubic eqn. Max of A1 if justification absent	
(b)(iii)	$AB = \pm (r_A \cos \theta_A - r_B \cos \theta_B)$	M1		OE method to find AB or AB^2 .	
				eg $AB = \frac{OB\sin(\theta_B - \theta_A)}{\sin\theta_A}$ OE single 'eqn'	
				л	
				or $AB^2 = r_A^2 + OB^2 - 2r_AOB\cos(\theta_B - \theta_A)$	
				or $OB^2 = r_A^2 + AB^2 - 2r_A AB\cos\theta_A$	
	$\cos \theta_B = \sqrt{\frac{r_B}{2}} \left(= \sqrt{\frac{\sqrt{13} + 1}{8}} \right) = (0.758(7))$	m1		OE eg solving correct quadratic	
	V 2 (V 8)			eg sin $\theta_B = \frac{3}{\sqrt{13} + 1}$ or $\theta_B = 0.709(41)$	
	AB = 0.425 (to 3sf)	A1	3	$\sqrt{13+1}$ 0.425 Condone >3sf (0.425428)	
	Total		18		
(b)(ii)	TOTAL		75		
	$(2\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 3) = 0 \sin\theta = 0.5 \text{ (pt } A\text{), eg } \sin\theta < -1 \text{ impossible, so } \sin\theta = \frac{-2 + \sqrt{52}}{8}$				